# A switching feedback control approach for the persistence of managed resources

Phoebe Smith[1], Chris Guiver[2], Stuart Townley[3] and Daniel Franco[4]

[1] Department of Mathematical Sciences, University of Bath; [2] School of Engineering & the Built Environment, Edinburgh Napier University; [3] Environment and Sustainability Institute, University of Exeter; [4] Departamento de Matemática Aplicada, Universidad Nacional de Educación a Distancia (UNED)

An extended version of this research has been accepted for publication, see [1]. For further information email: psjs22@bath.ac.uk

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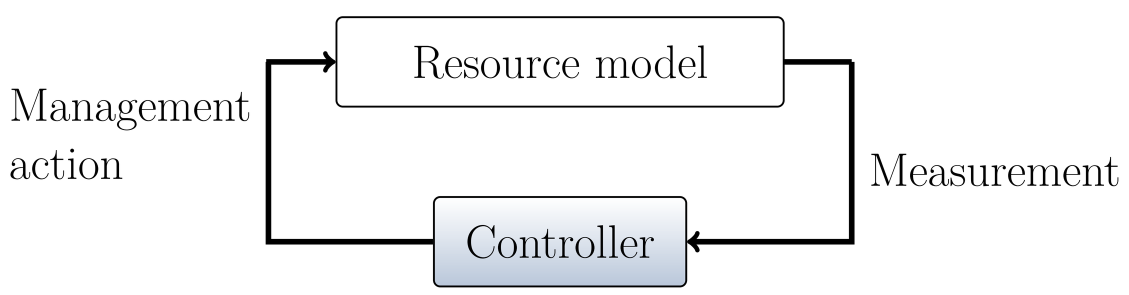
## *How to control a quantity which is poorly understood?*

## Problem Statement

- Design a novel switching function, inspired by adaptive control, that identifies a suitable strategy which gives desirable dynamic behaviour, assuming that such a strategy exists.
- Here, a desirable strategy corresponds to a strategy resulting in persistence of the population.
- *Assumptions*: there are at least two strategies to choose between and that at least one of these strategies corresponds to the population persisting.
- *Desired result*: measurements indicating a small population cause our model to switch strategy, whilst measurements indicating a large population cause no further switches and so the solution is reached.
- Mathematically, achieved using a feedback control approach and population measurements taken at discrete time intervals to inform the choice of strategy.




Block diagram of feedback interconnection

## Objectives

- The tool will identify a strategy corresponding to persistence from a choice of discrete and distinct control strategies available
- Applicable for use on species where the parameters of the population model are uncertain.

## Novelty

To avoid extinction, we seek to destabilise the zero-equilibrium. However, adaptive control typically seeks to stabilise the zero equilibrium of a control system. Hence, this work is novel in the application to resource management *and* in the theory developed.

## Model

Each management strategy corresponds to a positive difference equation

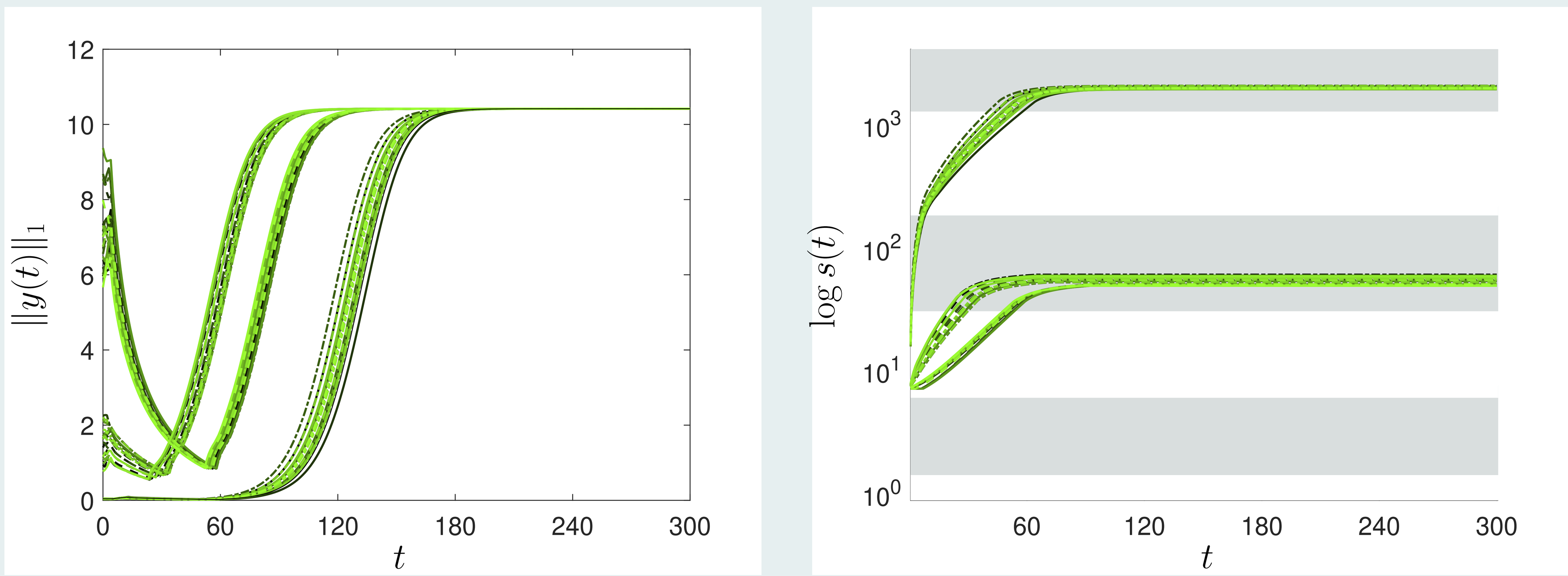
$$x(t+1) = F\big(r, x(t)\big), \quad x(0) = x_0\,. \tag{1}$$

Here, the population of interest, $x(t)$, varies temporally with discrete time-step $t$ and may be scalar- or vector-valued and the integer $r$ denotes which of the $q$ management strategies is applied at time $t$. For fixed $r$, the function $F(r,\cdot)$ describes the dynamics of $x$, with initial condition $x_0$, and may be linear or nonlinear. We use a feedback control approach and population measurements, $y(t)$, to design a switching system that identifies desirable strategies. The adaptive switching feedback control scheme is given by the system of positive difference equations

$$\left.\begin{aligned} x(t+1) &= F\big(\mathcal{K}(s(t)), x(t)\big), \quad x(0) = x_0\,, \\ s(t+1) &= s(t) + \begin{cases} 0\,, & M \le \|y(t)\|,\ \|y(t)\| = 0 \\ \dfrac{1}{\|y(t)\|} & \|y(t)\| < M\,, \end{cases} \quad s(0) = s_0\,, \end{aligned}\right\} \quad t \in \mathbb{Z}_+\,. \tag{2}$$

The control strategy applied at time $t$ is $\mathcal{K}(s(t)) \in \{1, \cdots, q\}$ and is determined by $s$ and a fixed sequence $\tau$ via the function $\mathcal{K} : \mathbb{R}_+ \to \{1, \cdots, q\}$. The switching sequence, $s$ is updated using the measured variable $y$ and the design parameters $M > 0$ and $s_0$.

## Numerical results




(a) Trajectories of the population size $\|x(t)\|_1$ against time.

(b) Semilog plot showing the growth of the $s$ sequence over time. The light grey and white shaded regions correspond to strategy 1 and 2, respectively.

Figure 1: Simulation outputs of the nonlinear population model of trout cod with one hundred random initial conditions.

- Density-dependent stage-structured population projection matrix model for female trout cod (*Maccullochella macquariensis*) [2], categorised as vulnerable by the IUCN Red List of Threatened Species [3].
- The model has annual time-steps and units corresponding to $10^3$ fish.
- *Model assumptions*: two management strategies available and only adult fish can be observed.
- Strategy 1 and 2 result in persistence and extinction of the population, respectively.
- Simulated for one hundred random initial conditions.
- For all initial conditions, the observed population size, $\|y\|_1$, settles to the equilibrium, that is persists (Figure 1a), whilst $s$ becomes bounded in the desirable strategy (Figure 1b).

## Theorem

Consider (2) where $F$ is as in (1) with $q \geq 2$. Under suitable assumptions, the following statements hold

- $s$ is bounded;
- $\mathcal{K}(s(t)) \to r$ as $t \to \infty$ where $r$ is a desirable strategy;
- $x$ persists as $t \to \infty$.

## Summary

- A novel and robust feedback control solution has been developed in the context of conserving populations where there are a discrete number of management options.
- The switching system uses per time-step population measurements to identify a management option that corresponds to the persistence of the population.
- The assumptions placed on the $F(r,\cdot)$ are structural and not restrictive in this, ecological, setting.
- Envisage the scheme being used in situations where the $F(r,\cdot)$ are unknown, or highly uncertain.

## Future Work

- Relax assumption that there is a desirable strategy by including assumptions regarding the transient nature of the models, so that extinction strategies can be coupled to give way to an persisting population overall, as seen in [4].
- Explore the middle ground between finding a robust and optimal solution.

## References

[1] D. Franco, C. Guiver, P. Smith, and S. Townley.
A switching feedback control approach for the persistence of managed resources.
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[2] C. R. Todd, S. J. Nicol, and J. D. Koehn.
Density-dependence uncertainty in population models for the conservation management of trout cod, maccullochella macquariensis.
*Ecological Modelling*, 171(4):359–380, 2004.

[3] J. Koehn, M. Lintermans, J. Lieschke, and D. Gilligan.
Maccullochella macquariensis.
*The IUCN Red List of Threatened Species*, 2019.
Available here.

[4] C. Guiver, D. Packman, and S. Townley.
A necessary condition for dispersal driven growth of populations with discrete patch dynamics.
*Journal of Theoretical Biology*, 424:11–25, 2017.